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TECHNICAL MEMORANDUMS

NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS

No. 705

THE CRITICAL SHEAR LOAD OF RECTANGULAR PLATES

By Edgar Seydel

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By Edgar Seydel

I. SCOPE OF PROBLEM

In the stress analysis of plates in shear the determination of the critical shear load, i.e., the load at which an originally flat plate begins to buckle, is of primary significance, because when this load is exceeded the plate breaks or at least assumes a different stress attitude (if the plate is very thin). The analytical method developed by Timoschenko** and Reissner*** for the critical load in shear has been extended to include the orthogonal-anisotropic (orthotropic) plate.****

The premises for the solution of the problem are: The perfectly flat, rectangular, thin, homogeneous plate of constant thickness is supported at the edges without being restrained. The edge supports are rigid and the connection of the plate with the edge supports is such that the plate along the four boundary lines cannot bend out of its plane. Along the four edges the evenly divided, shear load t of constant intensity, is applied, which is in outside equilibrium and which, provided the shear is so small that the plate remains flat, impresses the same shear stress in the entire plane of the plate. (See fig. 1.)

In view of the use of plywood plates and corrugated metal strip or other similarly stiffened plates, we shall

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*"Ausbeul-Schublast rechteckiger Platten." Z.F.M., February 14, 1933, pp. 78-83.

**Timoschenko: "Über die Stabilität versteifter Platten." Zeitschrift "Der Eisenbau," Vol. 12, Nos. 5 and 6, 1921, pp. 147-163.

***Bergmann and Reissner: "Über die Knickung von rechteckigen Platten bei Schubbeanspruchung." Z.F.M., Vol. 23, No. 1, 1932, pp. 6-12.

****"Über das Ausbeulen von rechteckigen, isotropen oder orthogonal-anisotropen Platten bei Schubbeanspruchung." (To be published in Ingenieur-Archiv, 1933.)

treat not only the isotropic but also the orthotropic plate in which the rigidity in two directions at right angles to each other is different. Herein it is assumed that these two directions (which, for instance, in the laminated plate correspond with the direction of the fibers in the outer and inner plies) are parallel to the plate edges.

II. FORMULAS FOR CALCULATING THE CRITICAL LOAD IN SHEAR OF RECTANGULAR PLATES

1. Isotropic Plate

Let a be the long side of the rectangular plate, b, the short side, and δ the thickness. Material properties as far as they enter into the discussion are characterized by the elasticity modulus E and the transverse elongation figure (Poisson's constant) ν. Then the plate stiffness is

$$D = \frac{E\ \delta^3}{12\ (1 - \nu^2)}$$

and the critical shear load is:

$$t_{kr} = c_a\ \frac{D}{(b/2)^2}$$

with c_a as a coefficient contingent upon aspect ratio

$\beta = b/a$ to be read from figure 2. For the infinitely long plate ($\beta = 0$), $c_a = 13.2$ (or more exact, 13.165); for the square plate ($\beta = 1$) it is $c_a = 23.0$; for other aspect ratios β, c_a lies between these two limits.

2. Orthogonal-Anisotropic Plate

Assume a and b are the two sides of the rectangular, orthogonal-anisotropic plate and δ the plate thickness; further, let the direction of side a be called x direction, and that of side b, y direction. (See fig. 1.)

Whereas the determination of the elastic form changes in the isotropic plate can be effected with the data on E and ν, it requires in the case of the orthotropic plate the data on E and ν for the directions given by the orthotropic anisotropy and in addition the data on shear modulus. Both E and ν are admittedly dependent on each other, so that one of these four quantities is already defined by the three others.

If E_x is the elasticity modulus for length changes of the fibers in the x direction and ν_x the corresponding transverse elongation factor, and further, if E_y is the elasticity modulus for length changes of fibers in the y direction and ν_y the corresponding transverse elongation factor, we have:*

$$\nu_y E_x = \nu_x E_y$$

Now let G be the shear modulus.

Then the stiffness per unit length is:

$$(EJ)_x = E_x \frac{\delta^3}{12} \quad \begin{cases}\text{flexural stiffness in x direction}\\ \text{(by flexure about y axis)}\end{cases}$$

$$(EJ)_y = E_y \frac{\delta^3}{12} \quad \begin{cases}\text{flexural stiffness in y direction}\\ \text{(by flexure about the x axis)}\end{cases}$$

$$4(GJ)_{xy} = 4G \frac{\delta^3}{12} \text{ torsional stiffness } \left(= G \frac{\delta^3}{3}\right)$$

Then we compute

$$D_1 = \frac{(EJ)_x}{1 - \nu_x \nu_y}$$

$$D_2 = \frac{(EJ)_y}{1 - \nu_x \nu_y}$$

$$D_3 = \frac{1}{2} (\nu_x D_2 + \nu_y D_1) + 2(GJ)_{xy}$$

$$= \nu_x D_2 + 2(GJ)_{xy} = \nu_y D_1 + 2(GJ)_{xy}$$

$$\frac{1}{\vartheta} = \frac{D_3}{\sqrt{D_1 D_2}} \quad \begin{cases}\text{(reciprocal criterion of the ortho-}\\ \text{tropic plate)}\end{cases}$$

If $\frac{1}{\vartheta} = 1$, we compute

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*Luftfahrtforschung, Vol. 8, No. 3, p. 78, or D.V.L. Yearbook, 1930, p. 242.

$$\beta_a = \frac{b}{a}\sqrt[4]{\frac{D_1}{D_2}} \quad (\leqq 1)^*$$

Then the critical shear load is:

$$t_{kr} = c_a \frac{\sqrt[4]{D_1 \; D_2{}^3}}{(b/2)^2}$$

The c_a factor in this formula is dependent on β_a and $1/\vartheta$ and may be taken from figure 3.**

But if $\frac{1}{\vartheta} > 1$, we figure with

$$\beta_b = \frac{b}{a}\sqrt{\frac{D_3}{D_2}}$$

and the critical shear load is:

$$t_{kr} = c_b \frac{\sqrt{D_2 \; D_3}}{(b/2)^2}$$

The definition of c_b, which occurs in this formula, is explained in section IV of the report cited in footnote **** (first page).

Plates with evenly disposed reenforcements can, unless the spacing is unduly great, be approximated as homogeneous orthotropic plates and $(EJ)_x$, $(EJ)_y$, and $4(GJ)_{xy}$ are then expressed by mean values. The method of computing this stiffness, say, for corrugated metal sheet, has been described in D.V.L. report No. 230, "Column Tests in Shear with Corrugated Metal Sheet." (D.V.L. Yearbook, 1931, pp. 233-245.)

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*If β_a should yield a figure greater than 1, the notations for the two rectangular sides should be exchanged, i.e., if the longer side was denoted by a, it should then be changed to b and vice versa. The same applies, of course, to D_1 and D_2. Ordinarily the long side is called a, and the short side b, but it may happen that the stiffness in direction of the long side exceeds that in direction of the short side, in which case then the switch must be made.

In this diagram we chose the reciprocal criterion $1/\vartheta$ in preference to ϑ as parameter, to ensure linear interpolation for any criteria (within range of $1 \leqq \vartheta \leqq \infty$). The curves c_a (β_a) were obtained by an approximation method. (See footnote ** (first page.) Although it is likely that the so calculated c_a values do not substantially deviate from the exact values.

III. EXAMPLES

Isotropic Plate

1. Example:

Duralumin plate, 0.2 mm thick.

$$E = 7.5 \times 10^5 \ \mathrm{kg/cm^2} \qquad \nu = 0.25$$

a) Dimensions: length 45 cm, width 15 cm

$$D = \frac{E\ \delta^3}{12\ (1 - \nu^2)} = \frac{7.5 \times 10^5 \times 0.02^3}{12\ (1 - 0.25^2)} = 0.533 \ \mathrm{kg/cm}$$

For $\beta = \frac{b}{a} = \frac{15}{45} = 0.333$, figure 2 gives:

$$c_a = 14.3,$$

that is, a critical shear load of

$$t_{kr} = c_a \frac{D}{(b/2)^2} = 14.3 \times \frac{0.533}{7.5^2}$$

$$= 0.136 \ \mathrm{kg/cm}$$

b) For an identical but square plate having sides $a = b = 15$ cm (i.e., $\beta = 1$), we have:

$$c_a = 23.0$$

and consequently,

$$t_{kr} = 23.0 \times \frac{0.533}{7.5^2} = 0.218 \ \mathrm{kg/cm}$$

Orthotropic Plate

2. Example:

Plywood plate, 2 mm thick. The elasticity modulus*

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*The elasticity factors really should be determined by flexure and torsion tests. See Hertel's report on the shear moduli of laminated and ply wood, D.V.L. Yearbook, 1932, III, pp. 43-52.

in the direction of the grain (i.e., of that of the outer ply) is assumed at 150,000 kg/cm², the corresponding transverse elongation factor at 0.4, the elasticity modulus across the grain (i.e., of that of the inside ply) only 15,000 kg/cm² (= one-tenth of 150,000), the corresponding transverse elongation factor likewise 0.04 (i.e., one-tenth of 0.4), and the shear modulus $G = 10{,}000$ kg/cm².

a) Dimensions of plate: 40 by 60 cm; direction of grain parallel to short (40 cm) side.

$$a = 60 \text{ cm} \qquad E_x = 15{,}000 \text{ kg/cm}^2 \qquad \nu_x = 0.04$$

$$b = 40 \text{ "} \qquad E_y = 150{,}000 \text{ "} \qquad \nu_y = 0.4$$

$$G = 10{,}000 \text{ "}$$

$$J = \frac{\delta^3}{12} = \frac{0.2^3}{12} = 0.667 \times 10^{-3} \text{ cm}^3,$$

$$D_1 = \frac{E_x}{1 - \nu_x \nu_y} J = \frac{15000}{0.984} J = 15{,}250 \; J \text{ kg/cm}^2,$$

$$D_2 = \frac{E_y}{1 - \nu_x \nu_y} J = \frac{150000}{0.984} J = 152{,}500 \; J \text{ kg/cm}^2 = 10 \; D_1,$$

$$D_3 = \nu_y D_1 + 2 \, G \, J = (6{,}100 + 2 \times 10{,}000) \; J = 26{,}100 \; J \text{ kg/cm}^2,$$

$$\frac{1}{\vartheta} = \frac{D_3}{\sqrt{D_1 D_2}} = \frac{26100}{15250\sqrt{10}} = 0.54,$$

$$\beta_a = \frac{b}{a}\sqrt[4]{\frac{D_1}{D_2}} = \frac{40}{60} \times \frac{1}{\sqrt[4]{10}} = 0.375$$

According to figure 3, $c_a = 11.8$

$$t_{kr} = c_a \frac{\sqrt[4]{D_1 \; D_2{}^3}}{(b/2)^2} = 11.8 \; \frac{15250 \sqrt[4]{10^3}}{20^2} \times 0.667 \times 10^{-3}$$

$$t_{kr} = 1.69 \text{ kg/cm}$$

b) Now assume that the plywood plate is divided into four equal parts by three strips running in the direction of the side, which is 40 cm long (the short side). These stiffening strips (as well as the edges) shall be so stiff as to prevent bending out of the plane of the plate when the latter buckles; the plate to be supported without fixity at the stiffening strips as well as at the edges. The grain to run parallel to the long (40 cm) side.

$$\frac{1}{\vartheta} = 0.54 \quad \text{(unchanged)}$$

$$\beta_a = \frac{15}{40}\sqrt[4]{10} = 0.667$$

According to figure 3:

$$c_a = 13.5$$

$$t_{kr} = 13.5 \, \frac{15250\sqrt[4]{10}}{7.5^2} \times 0.667 \times 10^{-3}$$

$$t_{kr} = 4.35 \text{ kg/cm}$$

The stiffening strips raise the critical shear load approximately 2.6 times.

c) The same example as a), but with grain running parallel to the 60 cm side.

E_x and E_y (D_1 and D_2) would then have to be switched, but then $\beta_a > 1$, so we put

$$\beta_a = \frac{60}{40}\,\frac{1}{\sqrt[4]{10}} = 0.842$$

According to figure 3: $c_a = 15.4$

$$t_{kr} = 15.4 \, \frac{15250\sqrt[4]{10^3}}{30^2} \times 0.667 \times 10^{-3}$$

$$t_{kr} = 0.98 \text{ kg/cm}$$

In this case the critical shear load amounts to 0.58 of the value of example a).

d) The same example as b), but grain as in example c) parallel to 15 cm (respectively, 60 cm) side.

$$\beta_a = \frac{15}{40} \frac{1}{\sqrt[4]{10}} = 0.21$$

$$c_a = 11.3$$

$$t_{kr} = 11.3 \frac{15250 \sqrt[4]{10^3}}{7.5^2} \times 0.667 \times 10^{-3}$$

$$t_{kr} = 11.5 \text{ kg/cm}$$

With the grain running in this direction, the stiffening strips augment the critical shear load approximately 11.7 times, by changing the direction of the grain against example b) to about 2.6 times.

3. Example:

Square, corrugated duralumin strip: $a = b = 90$ cm
Elasticity modulus of strip: $E = 750,000$ kg/cm^2
Transverse elongation: $\nu = 0.25$
Cross section of corrugated sheet: $2l = 3.0$ cm (length of corrugation)
$2f = 0.915$ cm (height of corrugation)
Thickness: $\delta = 0.039$ cm

Applying the formulas given in report No. 230 of the D.V.L. Yearbook, 1931, pp. 233-245 (table II and figures 5 and 6), we find:

$$D_1 = 3.27 \text{ kg/cm}$$

$$D_2 = 3,790 \text{ kg/cm}$$

$$D_3 = 3.585 \text{ kg/cm}$$

$$\vartheta = \frac{\sqrt{3.27 \times 3790}}{3.585} = 31.1; \quad \frac{1}{\vartheta} = 0.032$$

$$\beta_a = \frac{90}{90} \sqrt[4]{\frac{3.27}{3790}} = 0.171$$

$$c_a = 8.3$$

$$t_{kr} = 8.3 \frac{\sqrt[4]{3.27 \times 3790^3}}{45^2} = 2.7 \text{ kg/cm}$$

Although the plate is square the coefficient c_a is only about 2 percent higher than $c_a = 8.125$ resulting for an infinitely long orthotropic plate $(\beta_a = 0)$ with $\vartheta = \infty$.

IV. EXPERIMENTAL DATA

At the recommendation of and in collaboration with Professor Reissner, we made a series of shear tests on thin duralumin sheets (occasionally also with plates of German silver, tin and celluloid), in which the theoretical assumptions were to be fully corroborated. In order to ensure a free edge support in the experiment the horizontally rigged up plate was restrained on top and bottom side by knife-like edge rails. These rails formed a rectangle, one side of which was 15 cm long, whereas the other side was 1, 2, and 3 times as long. The test plate itself was slightly greater than the rectangle formed by the rails, extending about 1.5 cm beyond each side. About 0.5 cm distant and parallel to the rails we rigged up four loading rails hinged by bolts and provided with pins, spaced about 1.4 cm, which fitted the corresponding holes on the test plate. The loads which were mutually in equilibrium were applied at two diagonally opposite bolts of the hinged quadrangle, through which the loading evenly distributed, was initiated in the test plate. In order to disturb as little as possible the irregular elongations normal to the edge which result from buckling, the holes in the plates were elongated.

The buckling was recorded by a Zeiss dial gage. Since

NOTE.- The metric values used in this T.M. can be converted to the English units by use of the following conversion factors.

mm	×	.03937	=	in.
cm	×	.3937	=	in.
cm^3	×	.061023	=	cu.in.
kg	×	2.20462	=	lb.
kg/cm	×	5.59977	=	lb./in.
kg/cm^2	×	14.2235	=	lb./sq.in.

the pressure exerted by the spring inside of the clockwork would signify a minor but still appreciable bending load on the test plate, the spring tension was so balanced that the tip of the dial gage, hence the pointer in every position which the tip assumed, stood still owing to the internal friction of the dial gage, but upon slightly tapping, moved a little in the sense of the spring force effect. Prior to each reading the tip was first set slightly lower than the test point (metal plate), then the tapping was continued until the tip touched the test point. This contact was recorded by a circuit and a galvanometer deflection.

To illustrate the test results the deflections of the plate at a test point (a point at which a maximum buckling was anticipated, generally in the plate center) were plotted against the progressively increased loadings. In addition we plotted contour-line charts based upon the readings of a greater number of test stations for each load stage, which brought out the differences in the buckling at different load stages (generally the differences between a load beyond the theoretical buckling load and zero or very small load).

Theoretically the aspect of buckling at a test point when shown versus the load should be as the dot-dash line in figure 4. There is no sign of buckling as the load increases till the critical load is reached. Then buckling occurs suddenly, i.e., the curve should at its start (on the abscissa axis) have a tangent perpendicular to the abscissa axis. The experiments failed to reveal such a curve as shown in figures 4 to 7. Even the measured contour maps (figs. 8 to 10) show a not unsubstantial discrepancy from those obtained according to the theory.* Said discrepancies in theory and test reveal that the theoretical assumptions were not fully complied with in the experiments. In particular, in the tests the effect of even very small initial buckles appears perceptible, which could not be altogether avoided.

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*The theoretical buckling areas for plates of 1:1, 1:2, and 1:3 aspect ratio are given in report cited in footnote *** (first page). Subsequent but more accurate calculations revealed that the joints (w = 0) of the buckling surfaces should have, where they go over into the edge lines, tangents at right angles to them.

We likewise experimented with corrugated metal plates which, with certain approximation, may be considered as orthotropic plates. Several of these tests are described in the report cited, No. 230, D.V.L. Yearbook, 1931, pp. 233-245; other similar tests, not touched upon in this report, yielded substantially the same results. In these experiments made in connection with the theoretical treatment of infinitely long orthotropic plate strip the value for $\beta_a = b/a\sqrt{D_1/D_2}$ lies between 0.034 and 0.072. The validity, surmised at that time on the basis of simple considerations, to introduce the value c_a for such small β_a as afforded by $\beta_a = 0$ (infinitely long plate) is now proved by the c_a (β_a) curve in figure 3, at least for the freely supported plate. Admittedly the plate edges in these tests were (almost rigidly) restrained (divergent from the problem treated in the present report).

The set-up for these experiments with one or two square corrugated duralumin sheets was somewhat different. The sides of the sheets were 90 cm long. The sheets consisted of three strips riveted together and had (apart from slight deviations definable only by the most accurate measurement) the same sectional dimensions. Four duralumin tubes formed the four edges, to which, on each side, a sheet was riveted. In order to initiate the shear more evenly through the individual rivets into the sheets, the resultant load allocated to each edge support was applied in the center (instead of at the end).

The riveting of the corrugated sheet produces an elastic restraint on the edge supports, which, compared to the free support, effects an upward shift in critical load. The buckling at different points was measured in the median line of the sheet perpendicular to the direction of the wave ridge. Whereas the majority of points revealed from the start certain buckling, at first gradual, then increasing, the course of buckling at one point, versus the load, corresponded throughout to the course anticipated by the theory, as shown in figure 11. The critical load must lie between the figures for free edge support and those for rigid restraint. (The value for rigid restraint in the figure corresponds to that for the infinitely long plate, but might be only slightly too small for the test sheet, because $\beta_a = 0.171$.)

The first test actually revealed no appreciable buckling up to a certain load (critical shear load). This critical load lies in the zone imposed by the elastic edge restraint. Only minor permanent form changes remained after unloading, which, however, effected certain deviations in the course of the curve when the experiment was repeated (as likewise anticipated, according to theoretical considerations). When repeated after having been previously unloaded, this deviation became still greater. The sheet failed at 2.3 times the buckling load, which was determined from the course of this curve.

V. SUMMARY AND REFERENCE TO THE PRACTICAL ASPECT OF THE PROBLEM

The report adduces formulas for analyzing the critical shear load of a simply supported square, isotropic, or orthogonal anisotropic plate. Although arrived at by approximation method, these formulas should nevertheless be amply accurate for practical purposes, especially since the premises of the theory (unrestrained support at the edges, perfectly flat plate) never are precisely fulfilled. The experimental data reveal that in very thin plates the disturbing effect of incipient buckling is quite considerable.

If the plates are very long, rigid edge restraint can also be taken into consideration by means of the formulas given in the D.V.L. Yearbook for 1930, p. 237.

The interpretation of the strength of a plate in shear according to the ratio of plate thickness to plate width with a view to the problem in point here can be effected for three cases:

1) That the plate is so thick that failure occurs by exceeding the shear strength, while the plate remains flat without signs of buckling;

2) That the (pure shear) stresses in a thin plate up to attainment of critical shear load are relatively low. Owing to the buckling, however, due to this load, rapidly increasing flexural stresses are induced which may after exceeding the critical load lead to failure of the plate; in which case, the critical load is of primary significance for the strength.

3) That the flexural stresses contemporary with the

buckling are comparatively low in very thin plates, such as used in aircraft construction; that they remain (apart from harmless purely local excesses) below the yield limit, so that, finally, the break occurs by exceeding the tensile strength after forming the Wagner tension diagonal field. Starting with the critical load the elastic form changes occurring in the sense of the applied loads, increase in greater measure than before. Otherwise, the critical load has no significance for the evaluation of the strength.

Admittedly, no sharp line can be drawn between cases 2 and 3. For the bar stressed in compression, case 2 is identical with the Euler load (column failure), case 1, with a short thick column which fails before reaching the stability limit by exceeding the compressive strength of the material, and case 3, with a very slender bar which can be loaded until both ends meet and even extend beyond without inducing permanent form changes, although this case is of scarcely any practical import for the compressed bar, in contrast to the plate stressed in shear.

Translation by J. Vanier,
National Advisory Committee
for Aeronautics.

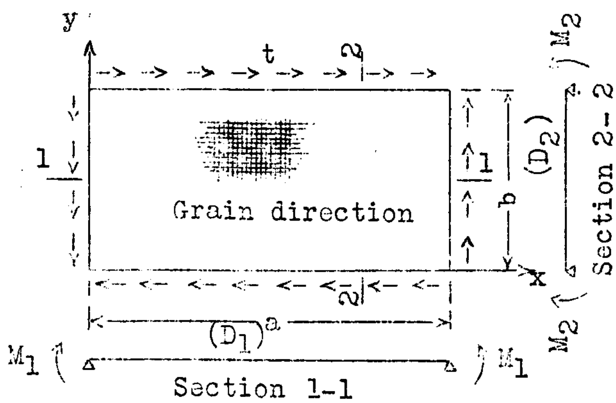

Figure 1.- Rectangular plate load in shear t, the grain direction running parallel to the plate edges (as, say, of a plywood plate) was used as basis in the treatmant of the orthogonal-anisotropic plate. For loading by moment M_1 the flexural stiffness D_1 (elasticity modulus E_x) is decisive, for loading by moment M_2 the flexural stiffness D_2(elasticity modulus E_y) is significant.

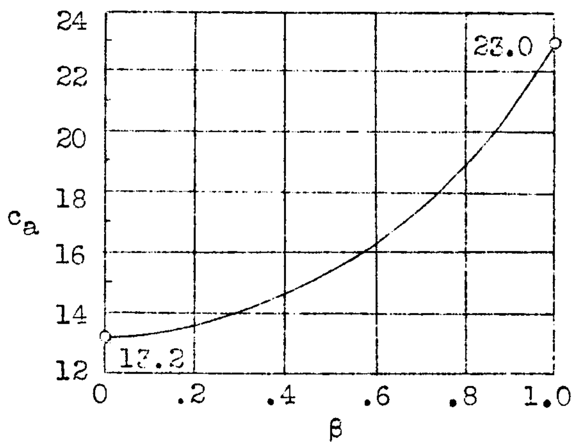

Figure 2.- Coefficient

$$c_a = \frac{t_{kr}\ (b/2)^2}{D}$$

of the critical shear load of an isotropic plate versus aspect ratio $\beta = b/a$

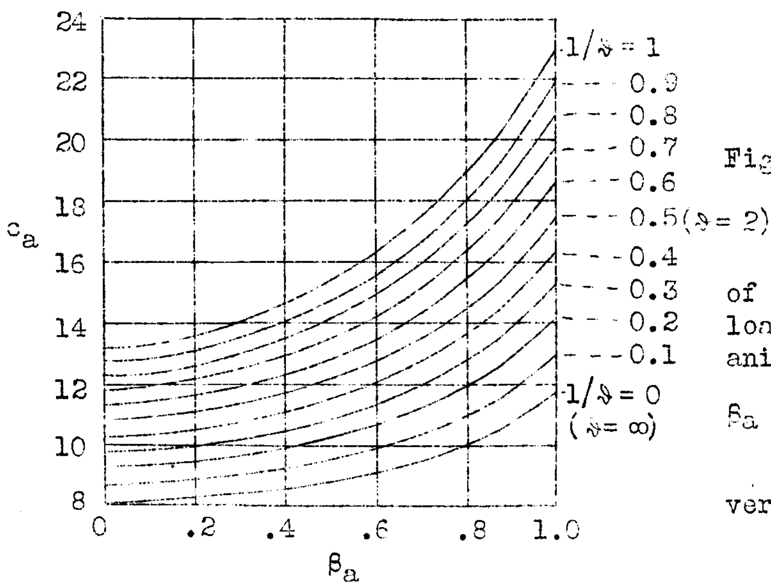

Figure 3.- Coefficient

$$c_a = \frac{t_{kr}\ (b/2)^2}{\sqrt[4]{D_1 D_2{}^3}}$$

of the critical shear load of an orthogonal-anisotropic plate versus

$$\beta_a = \frac{b}{a}\sqrt[4]{\frac{D_1}{D_2}} \text{ and}$$

versus $\frac{1}{\vartheta} = \frac{D_3}{\sqrt{D_1 D_2}}$

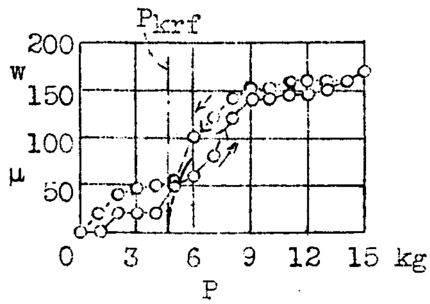

Figure 4. - Test data: buckling w (in 10^{-3} mm) versus load P (kg), recorded in center of 0.2 mm gage square dural plate (a = b = 15 cm) by loading and unloading. P and P_{krf} (theoretical critical load) are the resultant loads falling into the plate diagonals. The dot- dashed line shows the anticipated curve under ideal test conditions.

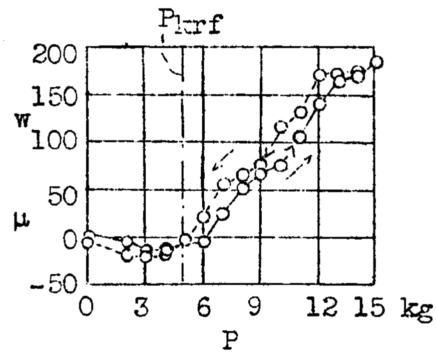

Figure 5.- Test data: same as in Figure 4, except that b/a = 1/2 (b = 15 cm, a = 30 cm)

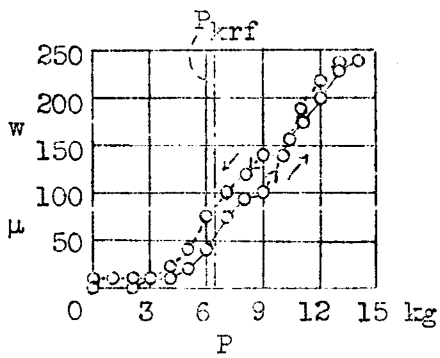

Figure 6. Test data: same as in Figure 4, except that b/a = 1/3 (b = 15 cm, a = 45 cm)

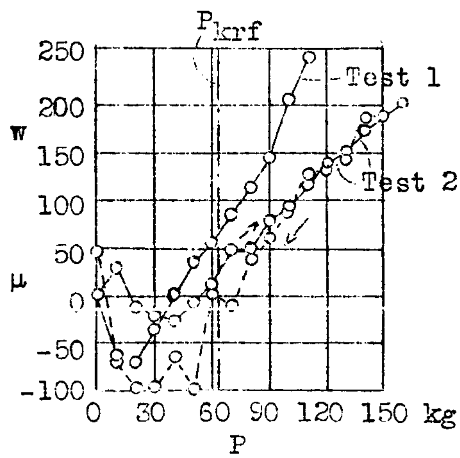

Figure 7.- Test data: same as in Figure 4, except plate thickness, 0.428 mm, b/a = 1/3 (b = 15 cm, a = 45 cm)

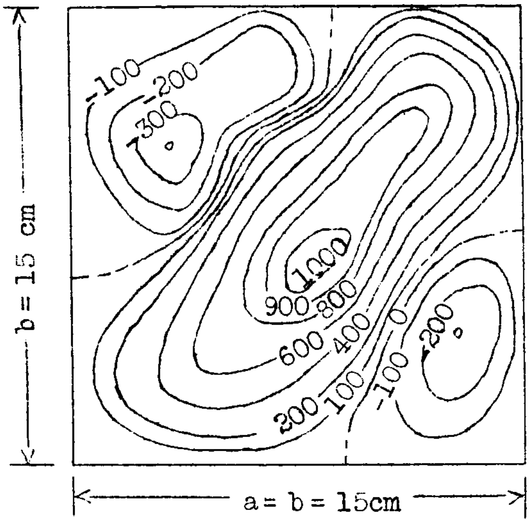

Figure 8.- Test data: contour lines of buckling surface of a 0.3 mm gage square dural plate at P = 50 kg (= 3.2 P_{krf})

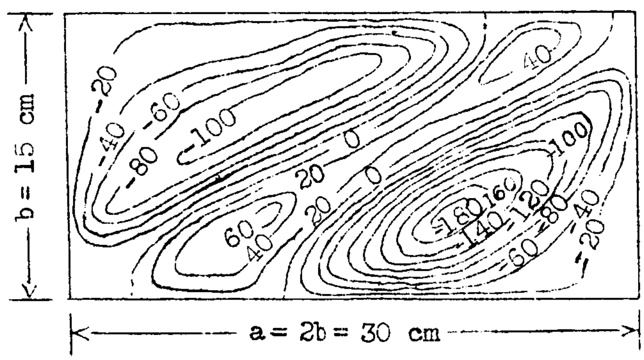

Figure 9.- Test data: contour map of a 0.2 mm gage dural plate (b/a = 1/2) by P = 42 kg (= 8.47 P_{krf}). (Discrepancy of buckling versus P = 2 kg)

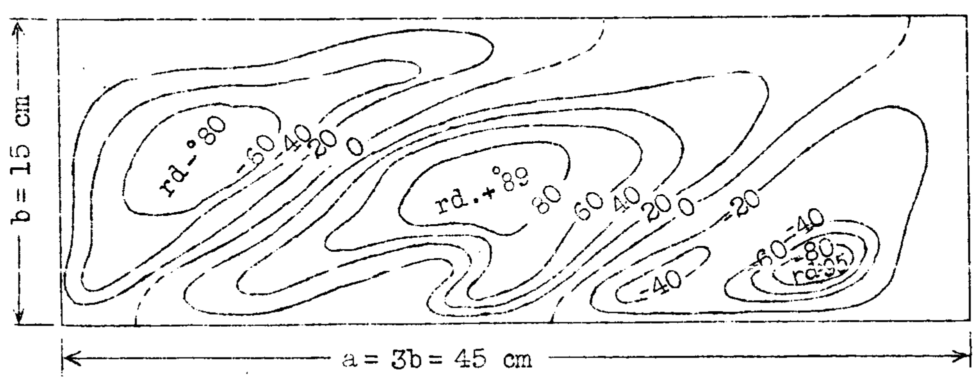

Figure 10.- Test data: contour map of a 0.2 mm gage dural plate (b/a = 1/3) by P = 30 kg. (= 4.67 P_{krf})

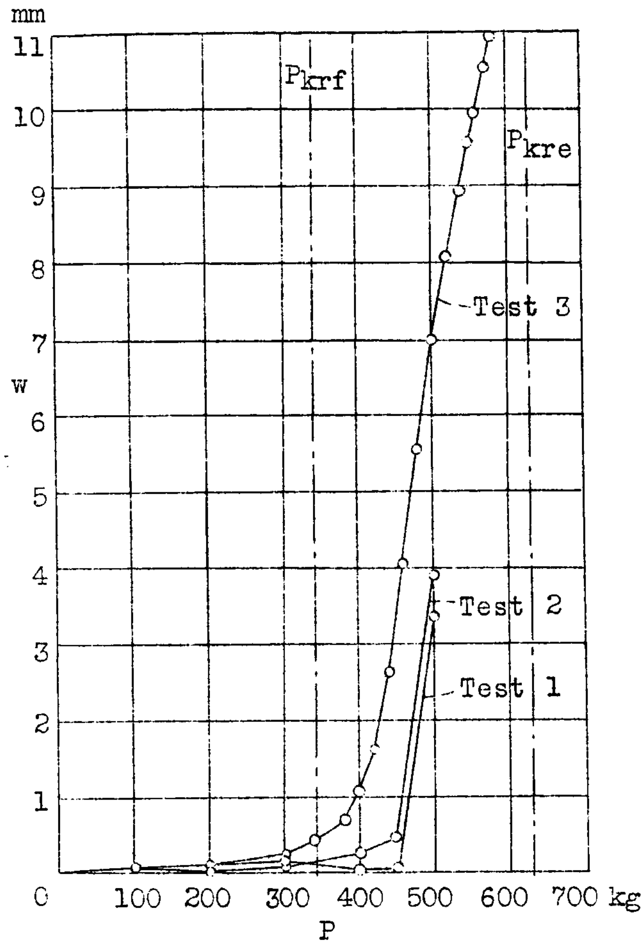

Figure 11.- Test data; buckling w versus load P (resultant load in diagonal direction), recorded on a square corrugated sheet (side = 90 cm section: corrugation heigth / corrugation length / sheet thickness (in mm) = 9.15/ 30, 0/ 0.39) by multiple loading. (P_{krf} and P_{kre}) denote the theoretical critical load by simple support and rigid plate edge restraint.